Lesson Research Proposal for **8th Grade Math -- *Solving Systems of Equations***

For the lesson on May 16, 2017
At Sherrard Junior High School, Kathy Felt's 8th Grade Math Class
**Instructor:** Kathy Felt
**Lesson plan developed by:** Amanda Carson, Heather Brown, Kathy Felt, Alfreida Jamison, Laura Kaplan, Kandace McCoy, Nicole Rogers, Jeanine Sheppard, and Rebecca Wattleworth

# 1. Title of the Lesson: The Colossal Cookie Calorie Caper - Using Systems of Equations

# 2. Brief description of the lesson

Students will use systems of equations to investigate the construction of a variety of Oreo Cookies. Students will compare original and double stuff cookies to determine the nutritional information in the stuffing and the wafer and use this to determine the total nutritional information in a triple double Oreo.

# 3. Research Theme

The lesson study group is focusing on Mathematical Practice Standard 3: Construct viable arguments and critique the reasoning of others. The goal is to get students to play a more active role in the learning and be more involved in discussion during the class. We hope to engage the students in a challenging task that can employ multiple solution paths then charge the groups with relating the mathematical reasoning of each method.

# 4. Goals of the Unit

This is the last unit of the year and is a preparation for the final exam. We are revisiting systems of equations because the students need stronger conceptual understanding of the topic.

# 5. Goals of the Lesson:

a) Students will persevere in solving the problem of how many calories are in a Triple Double Oreo Cookie using any mathematical method they are comfortable with and the nutritional information of a Classic Oreo and a Double Stuff Oreo.
b) Students will use whatever mathematical approach they are comfortable to find engage in the task as long as they can explain the mathematical reasoning behind their process.
c) Students will review the method they used to solve the problem against use of a system of equations and see how the methods compare.
d) Students will develop a deeper understanding of the mathematics behind solving a system of equations.

e) Students will evaluate the efficiency of the methods and determine when one method is more suitable than another.
d) Students will analyze nutritional information and use it in their problem solving.

# 6. Relationship of the Unit to the Standards

| Related prior learning standards | Learning standards for this unit | Related later learning standards |
|---|---|---|
| **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.<br>**7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.<br>**8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*. | **8.EE.8** Analyze and solve pairs of simultaneous linear equations.<br>Research Lesson: **8.EE.8c** Solve real-world and mathematical problems leading to two linear equations in two variables. | **HS.A-REI.C.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. |

# *7.* Background and Rationale

Our lesson theme focuses on Mathematical Practice Standard 3: Construct viable arguments and critique the reasoning of others. The topic of our lesson is Solving Systems of Equations. As the end of the year nears, it is noted that not only do 8th grade students struggle with the concept of solving linear systems, but they also lack

discussion strategies in order to solve those systems. Thus, our goal is not only to observe students critiquing strategies of others, but also observe them being more involved in discussion during class in order to more deeply understand the math behind the traditional algorithms for solving systems of linear equations (elimination and substitution). We believe that by using Oreo cookies as part of the lesson, students will be engaged, have fun, and see the usefulness of systems of equations in solving everyday problems.

# 8. Research and *Kyozaikenkyu*

We began our research by individually seeking out resources on teaching, reviewing, or applying systems of equations. Those deemed notable were shared via email with the group, then discussed as a group at our next meeting. Resources discussed included lesson plans from TeachersPayTeachers, Dan Meyers, Math Made Possible, Mr. Kraft Wikispaces, Kid Courses, and CCSS Math Tasks from the North Carolina Board of Ed. These resources gave us a broad view of the types of tasks being used for the purpose of teaching systems of equations. Some of the resources were focused primarily on building procedural fluency while others were focused on applications and conceptual understanding. We narrowed our research and studied two application tasks in particular: the Oreo Task (http://maccss.ncdpi.wikispaces.net/file/view/CCSSMathTasks-Grade8.pdf/460716114/CCSSMathTasks-Grade8.pdf) and the Solar Panel Task (https://spacemath.gsfc.nasa.gov/Modules/8Mod7Prob1.pdf). We decided against the solar panel task because of access to the materials and concerns that students wouldn't relate to the context. There were many variations of The Oreo Task to be found, each providing varying levels of structure for the students as well as several different properties of Oreos to study. We incorporated strategies we employ during Math Talks to engage students in discussion, facilitate mathematical discourse, and encourage struggling students. The problems we started with during our research were very structured and scaffolded. We strived to remove as much student support as possible to let them struggle with the math (much like a 3 Act Math Task).

# 9. Unit Plan

**This unit plan is unique in that our research lesson was at the very end of the school year. The class was revisiting a concept they had learned earlier in the year. It ended up being a "review mini unit."

| Lesson | Learning goal(s) and tasks |
|---|---|
| 1 | Goal: Review Solving Systems of Equations.<br>What are Systems? What are we trying to do with systems? What is a solution and what does it mean to s*olve* a system of equations? *How* do we solve a system of equations?<br><br>Task 1: Solve a system of equations by graphing.<br>Solve a system of equations by through substitution using the "Blob" method of solving systems.<br>Solve a system of equations using the linear elimination method.<br>Do an example with 4 pants and 3 shirts costs $177, 3 pants and 2 shirts costs $127.50. How much is a shirt? |
| 2 | **The research lesson:** Oreo Task |

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| 3 | Goal: …Conceptual understanding of systems of equations and real world application of systems of equations and solving a system. What does the solution mean in the real world context?<br><br>Task: …Make connections to the Oreo Task to the symbolic connections |

# 10. Design of the Unit and Lesson

The students in this class are 8th grade math students in the “regular” track. There are students of all ability levels including very high (formerly in the advanced math track), average levels, and low ability. There are two students with IEPs. We will be in the last full week of school for the year at the time of this lesson. These students were previously introduced to Systems of Equations in February. Overall, it was a struggle for most. In the previous Illinois Learning Standards, teaching of systems of equations was done in high school. With the new Illinois Learning Standards Incorporating the Common Core, the topic of systems of equations is introduced in eighth grade. The students in this class are improving with linear graphing concepts, but systems have been more complicated for them. This lesson was designed with this information in mind. It will be an application of systems, in a fun and meaningful context for them. Math really is everywhere! Students are reviewing this concept for their final, to be administered a few days after this lesson. Students will have reviewed procedural methods to solve systems of equations a few days prior to the lesson.

This lesson will be administered to the students as inquiry. Students will be working in cooperative learning groups. A problem will be given to them that they will try to figure out (systems is one way to solve it; perhaps the most efficient way) on their own with minimal teacher guidance. We will try to determine if they will try to solve the problem using systems (or what methods were used and to what success), and we have built in this method for them to consider if they do not think of it on their own. Then, they will have to determine if systems is a practical and efficient method to use for the problem. What are the advantages to using systems? We scaffolded a systems solution that “another student” has used for their consideration. They will not be able to use any “non-math” methods to solve this problem. We want the students solving this on their own and explaining their methods and reasoning.

# 11. Research lesson

| **Steps, Learning Activities Teacher’s Questions and Expected Student Reactions** | **Teacher Support** | **Assessment** |
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| Introduction:<br><br>On the board when students walk in:<br>“How do you eat an oreo?”<br><br>Play students the Oreo commercial:<br>https://www.youtube.com/watch?v=cYnLDxf9S0Y | Have Internet ready cued to the oreo commercial.<br>Have the oreo cookies for each group. Single serving packs for each group with 6 original cookies and 3 double stuff cookies.<br><br>Allow the students a minute to answer the question of how they eat an oreo after watching the video. | Are the students actively engaged and interested in the context? |
| Posing the Task<br><br>Board:<br>“My ideal would be a triple double. What would be the nutritional information of a triple double?”<br><br>  | Provide the cookies with the nutritional information included.<br>A triple double consists of three wafers and two single stuffing.<br><br>Ask the students do a Think, Ink, Pair, Share.<br>Give one minute to think about their strategy of how they are going to do this.<br>Write their ideas down for one minute.<br>Talk to their group for a minute.<br>Ask the students to state any assumptions that they made during their discussion with their group. | Do students understand the task? Do they have an idea of where to start?<br><br>Are students eager to solve the problem? |

| Anticipated student responses | *(All times where we see calories mentioned, students may look at a different nutritional fact.)* | Are students able to tackle the problem? |
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| S1: Why can't I look on the nutritional information of that box? | T1: This is a Ms. Felt's cookie. We don't have it with us. | At what points do students struggle? |
| S2: It's a cookie and two/thirds. I figured how much was in one cookie, then divided it by the three parts (two wafers and a stuffing) and then the triple double has five parts. (or some other direct ratio of 1 and a half, etc). | T2: No. How many stuffings are included? How many wafers? Are they proportional? Does a cup of carrots and a cup of ice cream have the same amount of calories. | With questioning does S2 recognize why this can't be proportional? |
| S3: I don't have any idea where to start. | T3: Can we look at just the calories? How do they relate to the cookies? (If more is needed, specifically ask them how many calories per wafer and how many calories per stuffing) | How much prompting is needed to get student S4a to continue on to the rest of the problem? |
| S4a: Well, I know that one original cookie is 1/6th of the package, so I can divide the calories by 6 and get it for one cookie.<br>For the double stuff just divide by 3. | T4: Great. What can you do from there? (If necessary, so how much is in one wafer and one stuffing?) | |
| S4b (correct but not using a system): Well, I know 2 wafers and 1 stuffing is 53.3<br>And I know 2 wafers and 2 stuffings is 73.3.<br>Then it's only one stuffing different, so a regular stuffing would be 20 | T4b: Great, keep going, figure out how much is in the triple double.<br>If finished: How could you write this symbolically? | How are students interacting within the group, especially if the people in their group are solving it more than one way? |
| S5: (correct) well, since we did systems yesterday, this is kind of like that problem.<br>12w + 6 s =320<br>6w + 6s = 220 | | How do the different skill levels within a group affect the dynamic of the discussion? |
| S5a: (correct) so I do elimination and subtract and get 6w=100 calories and then substitute back | | Did students solve this in an unexpected |

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| S5b: (correct) 6s = 220-6w, so I can put that in the other equation for 6s.<br><br>S5c: (correct) If I solve the equation for w (or s), I get w=220/6-s<br><br>S5d: students have a system but make procedural errors.<br><br>S6: Recognize it as a system, but incorrectly create the system, such as<br>6w+12s=220 for double stuff<br><br>S7: Used one of the previous methods to find that a stuffing is 20 calories and a wafer is 16 ⅔ calories and found that a triple double would be 2(20)+3(16 ⅔ )= 90 (Depending on rounding, the students could end up a little more or less than this). | T5d: Check your work within the original cookies.<br><br>T6: Sketch a picture of the number of cookies with wafers and stuffing in the box. (If need be, let them open the actual package.) | way? |

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| Comparing and Discussing<br><br>S2 (recognizing why it can't be proportional.)<br><br>S5:<br>5d (students recognize the system, but know their answer isn't exactly working)<br>5c (get's correct answer but has to deal with a lot of fractions.)<br>5b (also substitution like 5c, but deals with fewer fractions)<br>5a (elimination is what this problem is "setup" for.)<br><br>S4a: providing the idea of doing it using reasoning without symbolic algebra.<br>S4b: completes the reasoning method. | Have a fake version of S2 available if the students in any one class do not do this method. Make sure the students understand why this is not a proportional relationship.<br><br>Have a fake version of S5a available if the students in any one class do not do this method.<br><br>Have a fake version of S4b available if the students in any one class do not do this method. Have a fake version of the algebraic representation of S4b ready to have students step through and explain the steps.<br>S4b equations:<br>12w+6s=320 Divide by 6 cookies<br>2w+s=53.33333 (calories in 1 original cookie)<br><br>6w+6s=220 Divide by 3 cookies<br>2w+2s=73.33333 (calories in 1 doublestuff)<br><br>2w+2s=73.33333 (calories in 1 doublestuff)<br>2w+s=53.33333 (calories in 1 original cookie)<br><br>s=20 (find difference tells me one stuffing)<br>Or: 2w+2s-(2w+s)=73.33-53.333 (find difference), s=20<br><br>2w+20=53.333<br>2w=33.333<br>w=16.667 | Do students recognize why a certain solution is incorrect, or appreciate the merits of one solution over another?<br><br>Are students supportive, listening, and able to question their classmates to push either their own or others' thinking? |
| Summing up<br><br>If we wrote an equation about one of our cookies, how would the equation have changed if we were talking about two cookies, or two packs of cookies?<br><br>Why are we "allowed" to do the steps in elimination and how do the steps in the cookie problem help us to understand that?<br><br>In what other situations would | Discuss these questions with the class if time. If no time, ask students to write a summary of these in their journals for homework and wrap up the following day. | Does the summary accurately represent the students' view of the lesson? |

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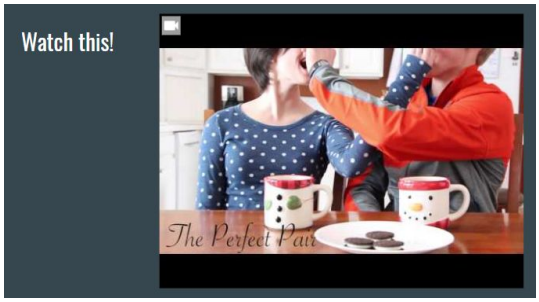
| this method be useful? | | |

# 12. Evaluation

a) Did the lesson successfully promote student-to-student discussion?
b) Did students recognize the need for a system of equations?
c) If students did not use systems of equations, did they recognize the connections after?
d) Did students stay actively engaged during the lesson?

# 13. Board Plan






How many calories are in a "triple double?"
Show all your work on the post-it chart paper.

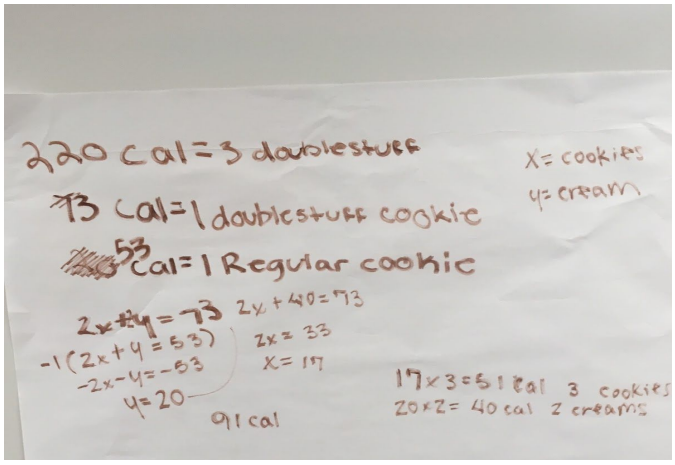
# 14. Reflection

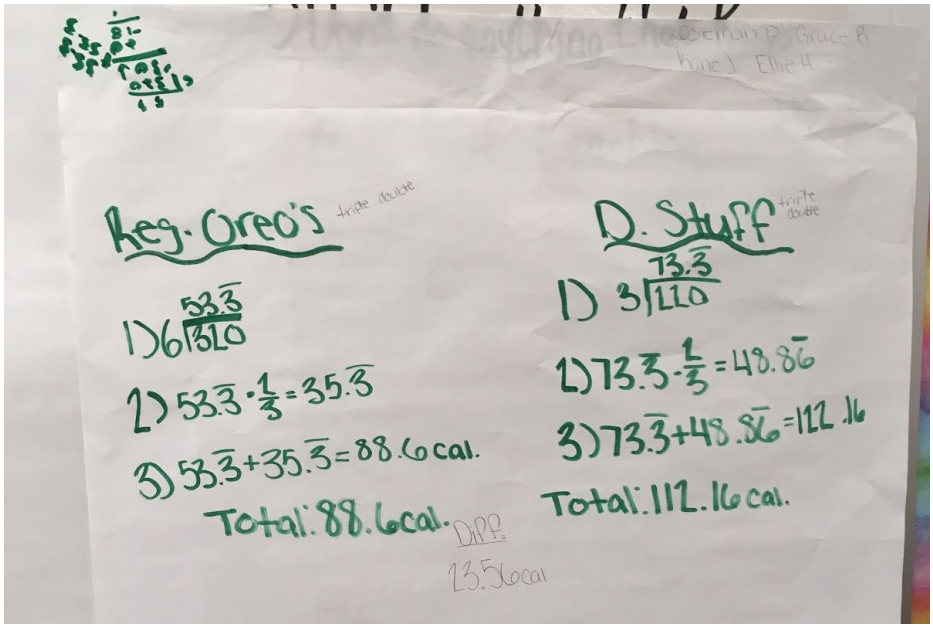
Before the research lesson, one of the team members taught a beginning version of the lesson:
When students first walked in the classroom, I had a picture of “How do you eat your Oreo.” I then showed the Oreo commercial to the students. They thought that they were going to be just answering how they ate their Oreo and maybe finding some area for how much they ate. When I posed the question how many calories would a triple double have in it, they jumped right in. None of the students started with systems of equations, but started with finding how much calories are in one cookie. I had several students who started this way. By the end of the hour, I had a few students who were able to find the calories in the wafer and the stuffing by finding the difference.

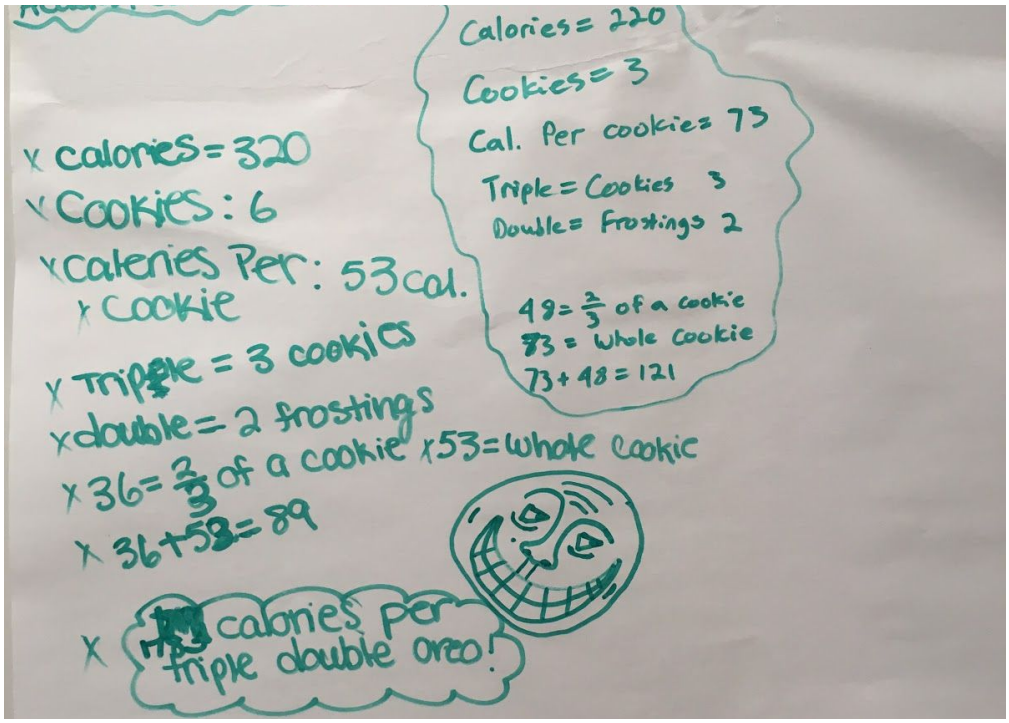
- Suggestion at end have the students put all the posters up and have the students look at the connections between them and think about the differences.
- Add questions for the teacher to ask the students to get them thinking without directing them too much.
- All of the kids first found out how much was in one original cookie and one double stuff cookie. Then most the kids tried to do proportional and then half the kids moved away from that to subtract to find the cream.
- Most of the kids had the stuffing and the wafer had the same amount of calories.
- Group dynamics - that some kids took over and some groups worked collaboratively. How could we get more to the collaborative side.
- Kathy noted that the students were very engaged by the video and the context.
- Needed more connection to the systems of equations. Have the students find another nutritional information as an exit slip or homework. (Would students see the efficiency of the systems of equations and choose to solve with it.)
- Assumptions that students made guided their work, but many of them didn’t realize how their assumptions were affecting the problem. (Even as small as rounding).
- 1 ⅔ vs 1.5 as most common answer change.
- The confusion between single stuffing and double stuffing in the triple double.
- One group was able to get to systems with minimal prompting.
- Wrap up could have been more structured (time was an element), but one group did say they are going first, and they had the correct answer.
- A use of a timer might have helped the lesson with a stopping point for the students.

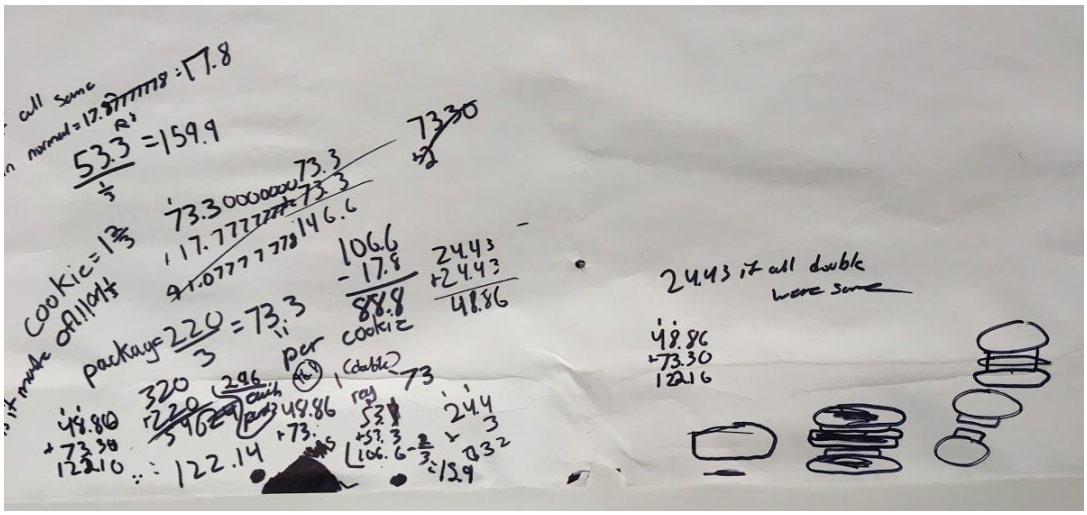
Student Reflections from the Research Lesson on Entrance Tickets the Following Day:
1. They did not like the extra teachers in the room but understood why they were there
2. They loved the oreos and figuring out how many calories were in the triple double oreo.
3. They loved the different ways to solve problems and loved seeing that you could use different ways to solve this problem too.
4. They liked working with their peers in groups to solve the problem. They were not sure they could have done it on their own. They liked sharing ideas and using other's ideas to get more ideas of their own.
5. They liked the video.
6. They liked the lesson and especially liked seeing at the end that systems could be used so easily to solve this problem.
7. They realized how to "make a complex problem simple."
8. They would have liked to have had one teacher per group if there were so many teachers here.

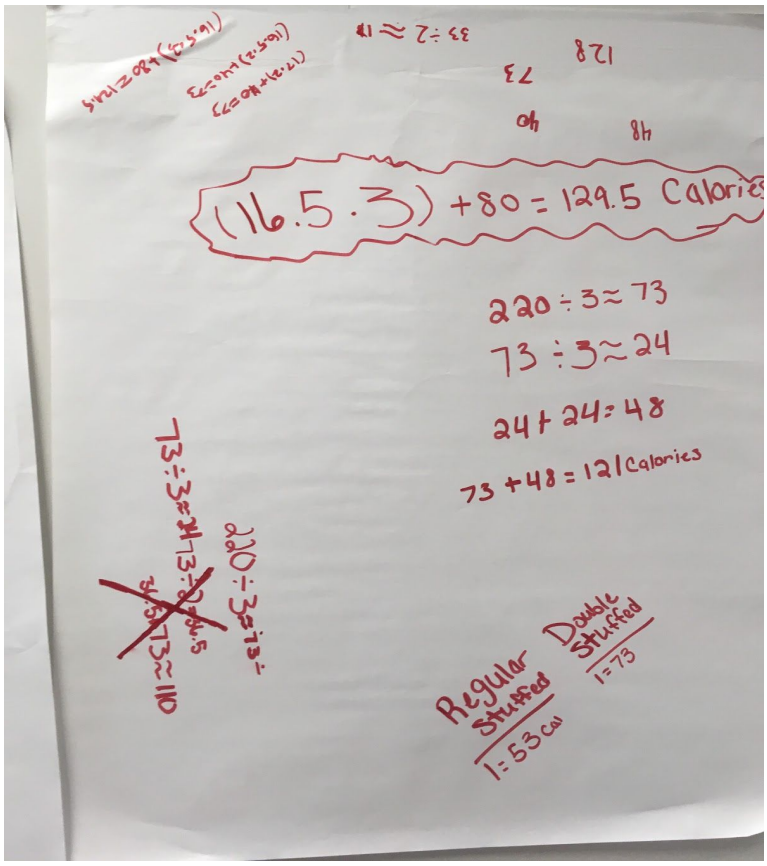
9. They wanted more clues
10. They liked how some people completely changed their attitudes and how they reacted to each other to solve the problem.
11. The "math was simple if you looked at it and didn't overthink it" (from a member of the group who didn't solve the problem).
12. They liked the concept and the idea.
13. They liked the real life problem.
14. Some didn't like having to make their thinking into an equation.

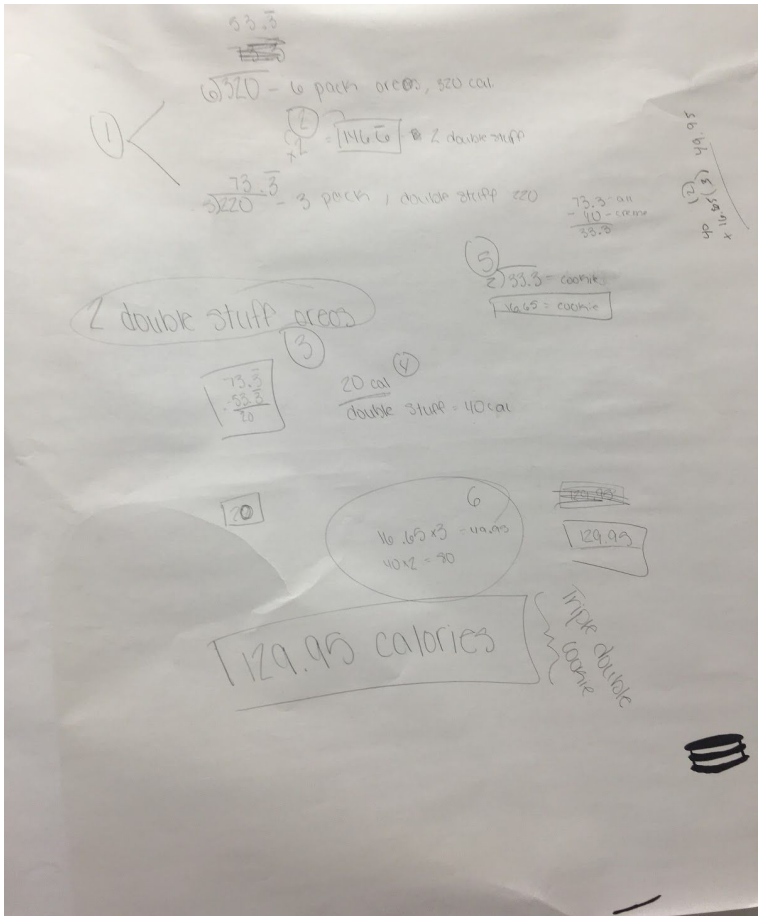



Calories = 220
Cookies = 3
Cal. Per cookie = 73
Triple = Cookies 3
Double = Frostings 2
49 = 2/3 of a cookie
73 = Whole Cookie
73 + 49 = 121
x calories = 320
x Cookies: 6
x Calories Per: 53 cal.
x Cookie
x Triple = 3 cookies
x double = 2 frostings
x 36 = 3/3 of a cookie
x 53 = whole cookie
x 36 + 53 = 89
x calories per triple double oreo!


53.3/3 = 159.9
73.30
106.6 - 17.8 = 88.8
24.43 + 24.43 = 48.86
24.43 if all double were same
48.86 + 73.30 = 122.16
320 - 220 = 100
122.14


(16.5 · 3) + 80 = 129.5 Calories
220 ÷ 3 ≈ 73
73 ÷ 3 ≈ 24
24 + 24 = 48
73 + 48 = 121 Calories
Regular Stuffed
1 = 53 cal
Double Stuffed
1 = 73


6)320 - 6 packs are 0, 320 cal
146.6 2 double stuff
3)220 - 3 packs / double stuff 220
2 double stuff oreos
20 cal
double stuff = 40 cal
16.65 x 3 = 49.95
40 x 2 = 80
129.95 calories
triple double cookie